Erratum: Green kernel estimates and the full Martin boundary for random walks on lamplighter groups and Diestel-Leader graphs

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In our paper in Ann. I. H. Poincaré - PR 41 (2005), 1101–1123, there is a mistake in the proof of the key Proposition 4.10: the use of dominated convergence on page 1112, line 5 from the bottom, is not justified since the dominating terms also vary when passing to the limit. Here is a correct proof that the error term $R(\mathfrak{d}_1, \mathfrak{d}_2)$ of (4.11) tends to 0 when $\mathfrak{d}_1$ is arbitrary (fixed) and $\mathfrak{d}_2 \to \infty$. (See Fig. 5 on page 1111 for a quick understanding of the involved quantities.)

Proof of Proposition 4.10.
Applying (3.1) to the projection π_1 gives $G_1(x_1, y_1) = \sum_{w_2 \in H(y_2)} G(x, y_1 w_2)$.

Let $w_2 \in H(y_2)$, where $H(y_2)$ is the horocycle of y_2 in $\mathbb{T}_r$. We write $v_2 = v(w_2)$ for the unique element in $H(x_2)$ that satisfies $v_2 \preccurlyeq w_2$. By Lemma 4.4, the random walk has to pass through some point of the form in $\{u_1 v_2 : u_1 \in H(x_1)\}$ on the way from x to $y_1 w_2$, and it also has to pass through some point in $\{c_1 u_2 : u_2 \in \mathbb{T}_r\,,\ \mathfrak{h}(u_2) = -\mathfrak{h}(c_1)\}$. Therefore, the stopping time $\mathbf{t} = \min\{\mathbf{t}_1(c_1)\,,\ \mathbf{t}_2(v(w_2))\}$ is a.s. finite, and the random walk passes through $Z_\mathbf{t}$ before reaching $y_1 w_2$. We obtain the decomposition (modified with respect to the old one)

$$\begin{aligned} G(x, y_1 w_2) &= \mathsf{E}_x\Big(G\big(Z_\mathbf{t}, y_1 w_2\big)\Big) \\ &= \mathsf{E}_x\Big(\mathbf{1}_{[\mathbf{t}_2(v_2) < \mathbf{t}_1(c_1)]}\, G\big(Z_{\mathbf{t}_2(v_2)}, y_1 w_2\big)\Big) + \mathsf{E}_x\Big(\mathbf{1}_{[\mathbf{t}_1(c_1) < \mathbf{t}_2(v_2)]}\, G\big(Z_{\mathbf{t}_1(c_1)}, y_1 w_2\big)\Big)\,. \end{aligned}$$

Now, if starting at x, we have $\mathbf{t}_2(v_2) < \mathbf{t}_1(c_1)$, then $Z_{\mathbf{t}_2(v_2)} = u_1 v_2$ for some random $u_1 \in H(x_1)$ that must satisfy $\mathfrak{u}(u_1, y_1) = \mathfrak{u}_1$ and $\mathfrak{d}(u_1, y_1) = \mathfrak{d}_1$, since c_1 cannot lie on $\overline{x_1\, u_1}$. But we also have $\mathfrak{u}(v_2, w_2) = \mathfrak{u}_2 = 0$ and $\mathfrak{d}(v_2, w_2) = \mathfrak{d}_2$. That is, the points $u_1 v_2$ and $y_1 w_2$ have the same relative position as the points x and y, and therefore $G(u_1 v_2, y_1 w_2) = G(x, y)$ by Lemma (4.3). We get

$$\mathsf{E}_x\Big(\mathbf{1}_{[\mathbf{t}_2(v_2) < \mathbf{t}_1(c_1)]}\, G\big(Z_{\mathbf{t}_2(v_2)}, y_1 w_2\big)\Big) = \mathsf{Pr}_x[\mathbf{t}_2(v_2) < \mathbf{t}_1(c_1)]\, G(x, y)\,.$$

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Now, given $v_2 \in H(x_2)$, there are precisely $r^{\mathfrak{d}_2}$ elements $w_2 \in H(y_2)$ with $v(w_2) = v_2$. Combining all these observations,

$$G_1(x_1,y_1) = \left(\sum_{v_2 \in H(x_2)} \mathsf{Pr}_x[\mathbf{t}_2(v_2) < \mathbf{t}_1(c_1)] \right) r^{\mathfrak{d}_2}\, G(x,y) + R(\mathfrak{d}_1,\mathfrak{d}_2)\,, \quad \text{where}$$

$$R(\mathfrak{d}_1,\mathfrak{d}_2) = \sum_{w_2 \in H(y_2)} \mathsf{E}_x\Big(\mathbf{1}_{[\mathbf{t}_1(c_1) < \mathbf{t}_2(v(w_2))]}\, G\big(Z_{\mathbf{t}_1(c_1)}, y_1 w_2\big)\Big)\,.$$

Let us first consider the error term.

$$\begin{aligned} R(\mathfrak{d}_1,\mathfrak{d}_2) &= \mathsf{E}_x \left(\sum_{w_2 \in H(y_2)} \mathbf{1}_{[\mathbf{t}_1(c_1) < \mathbf{t}_2(v(w_2))]}\, G\big(c_1 Z^2_{\mathbf{t}_1(c_1)}, y_1 w_2\big) \right) \\ &\le \mathsf{E}_x \left(\sum_{w_2 \in H(y_2)\,:\, d(w_2, Z^2_{\mathbf{t}_1(c_1)}) \ge \mathfrak{d}_1 + 2\mathfrak{d}_2} G\big(c_1 Z^2_{\mathbf{t}_1(c_1)}, y_1 w_2\big) \right)\,, \end{aligned}$$

since $\mathbf{t}_1(c_1) < \mathbf{t}_2(v(w_2))$ implies that $d(w_2, Z^2_{\mathbf{t}_1(c_1)}) \ge \mathfrak{d}_1 + 2\mathfrak{d}_2$ for the distance in $\mathbb{T}_r$ (look at Figure 5 !). Now observe that by Lemma 4.3, for any $k \ge 0$, the sum

$$\sum_{w_2 \in H(y_2)\,:\, d(w_2,z_2) \ge k} G(c_1 z_2, y_1 w_2)$$

depends only on $\mathfrak{d}_1$ and k, and not on the specific choice of $z_2 \in \mathbb{T}_r$ with $\mathfrak{h}(z_2) = -\mathfrak{h}(c_1)$. Therefore, choosing one such z_2, we get

$$R(\mathfrak{d}_1,\mathfrak{d}_2) \le \sum_{w_2 \in H(y_2)\,:\, d(w_2,z_2) \ge \mathfrak{d}_1 + 2\mathfrak{d}_2} G(c_1 z_2, y_1 w_2)\,.$$

Since $\mathfrak{d}_1$ is fixed, we can (again by Lemma 4.3) consider y_1 and c_1 as fixed points in $\mathbb{T}_q$ and move x_1 when $\mathfrak{d}_2 \to \infty$. But then the last sum is a remainder of the series

$$\sum_{w_2 \in H(y_2)} G(c_1 z_2, y_1 w_2) = G_1(c_1, y_1) < \infty\,.$$

Therefore $R(\mathfrak{d}_1,\mathfrak{d}_2) \to 0$ for fixed $\mathfrak{d}_1$, as $\mathfrak{d}_2 \to \infty$.

The rest of the proof remains unchanged. □

We remark here that *a posteriori*, $R(\mathfrak{d}_1,\mathfrak{d}_2) \to 0$ uniformly in $\mathfrak{d}_1$, as $\mathfrak{d}_2 \to \infty$. Indeed, when $\mathfrak{d}_1$ is large then $R(\mathfrak{d}_1,\mathfrak{d}_2) \le G_1(c_1, y_1)$ is small by formula (3.5).